Note 1

Introduction
The **mechatronics** field consists of the synergistic integration of three distinct traditional engineering fields for the system level design process. There three fields are:

1. Mechanical engineering, where the word ‘mecha’ is taken from
2. Electrical and electronics engineering, where the part of the word ‘tronics’ is taken from
3. Computer science

The mechatronics field is not simply the sum of these three major areas, but rather the field defined as the intersection of these areas when taken in the context of systems design, as shown in Figure 1. It is the current state of evolutionary change of the engineering fields that deals with the design of controlled electromechanical systems. Virtually every modern electromechanical system has an embedded computer controller. Therefore, computer hardware and software issues (in terms of their application to the control of electromechanical systems) are part of the field of mechatronics. Had it not been the widespread availability of the low-cost microcontrollers for the mass market, the field of mechatronics as we know it today would not have existed. The availability of embedded microprocessor for the mass market at an ever-reducing cost and increasing performance makes possible the use of computer control in thousands of consumer products.

Figure 1: The field of mechatronics: intersection of mechanical, electrical, and computer science.
The old model for an electromechanical product design team includes:

1. Engineer(s) who design the mechanical component of a product
2. Engineer(s) who designs the electrical components such as actuators, sensors, and amplifiers, as well as design the control logic and algorithms
3. Engineer(s) who design the computer hardware and software implementation to control the product in real time

A mechatronics engineer is trained to do all of these three tasks. In addition, the design process is not sequential from mechanical design, followed by electrical and computer control system designs, but rather all aspects (mechanical, electrical, and computer control) of design are done simultaneously for optimal product design. Clearly, mechatronics is not a new engineering discipline, but is rather the current state of the evolutionary process of engineering disciplines needed in design of electromechanical systems. **The end product of a mechatronics engineer’s work is a working prototype of an embedded computer-controlled electromechanical device or system.**

The analogy between a human-controlled system and a computer-controlled system is shown in Figure 2. If a process is controlled and powered by a human operator, the operator observes the behavior of the system (i.e., using visual observation), makes a decision regarding what action to take, and then, using his/her muscular power, a particular control action is taken. One could view the outcome of a decision-making process as a low-power amplified version of the control (or decision) signal. The same functionalities of a system can be automated by use of a digital computer as shown in the same figure.

The sensors replace the eyes, actuators replace the muscles, and the computer replaces the human brain. Every computer-controlled system has the following four basic functional blocks.

1. Process or plant to be controlled.
2. Actuators (e.g. pressure and position sensors)
3. Sensors (e.g. hydraulic and electric power actuators)
4. Controller (i.e., digital computer)
Figure 2: Manual and automatic control system analogy: (a) human controlled and (b) computer controlled.
Mechatronics design for a programmable closed-loop mechanism

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Abstract: As the demand increases for machines with high accuracy, high speed and high stiffness, programmable closed-loop linkages (PCLL) emerge. This paper presents further results obtained from a study of the mechatronics design approach to PCLL systems proposed by the authors elsewhere. In this approach, the system performances such as motion tracking and torque fluctuation are further improved after a suitable design of mass redistribution. In the present paper it is shown that a scheme called negative mass redistribution, which follows the principle of shaking force/shaking moment balancing, can achieve an excellent improvement in system performance. Furthermore, simultaneous variation in the length of the link and the gain of the PD controller is studied, which shows promise for further improvement in system performance. In general, these studies have shown that complex control algorithms may not achieve a better result than that achieved by a simple PD controller combined with a mass redistribution scheme.

Keywords: programmable system, closed-loop mechanism, design, control, mass redistribution

NOTATION

A(θi) inertia term
C damping coefficient
D dissipation energy
g gravity constant
I control performance index
Ji moment of inertia of link i
k spring torsional constant
K kinetic energy
Kd control gain for the derivative term
Kp control gain for the proportional term
Li length of link i
mi mass of link i
M summation of the input torque and the moment of the ground bearing forces
P potential energy
Pg potential energy due to gravity
Ps potential energy stored in the torsional spring
ri alternative radial distance of the mass centre in the local coordinate system
ri' alternative radial distance of the mass centre in the local coordinate system
Si mass centre of link i

1 INTRODUCTION

In most machinery design, closed-loop planar linkages are usually synthesized to achieve a specific motion task such as path generation or rigid body guidance [1]. Only a few studies of the motion tracking performance of closed-loop linkages have been reported in the literature so far [2-4]. (It is noted that, in this paper, motion...
tracking is defined as a smooth and continuous position and velocity tracking.) Generally, motion tracking tasks have been realized by means of programmable open-loop type mechanisms, i.e. robot manipulators. The open-chain kinematics structure, however, creates several design difficulties. For example, the positioning accuracy at the end-point of a long robot arm is considerably low; a small amount of error at each revolute joint is magnified at the end-point of the arm as its length gets longer; most importantly, the mechanical stiffness of the open-loop construction is inherently poor. As a result, the accuracy of the motion tracking performance will deteriorate. The research trend in modern machinery development therefore shifts towards the design of a new-generation mechanism, i.e. programmable, servomotor-driven, closed-loop linkages, for position and velocity tracking purposes [2–8].

The dynamics of a traditional closed-loop linkage is usually highly non-linear and complex owing to the asymmetry of its geometrical structure. This presents difficulties to control engineers in designing a controller to make a closed-loop linkage follow a trajectory precisely at high speeds. Several methods reported in the literature are proposed to handle the difficulties. As suggested by Lin and Chen [3], a very sophisticated control structure that is composed of several subcontrol algorithms such as a model reference adaptive control (MRAC), a disturbance compensation loop and a modified switching controller, plus some feedback loops, is proposed to control the linkage. In Youcef-Toumi’s works [7, 8], however, a different design strategy is adopted. With the aim of simplifying the dynamic model of the overall mechanical structure, a parallelogram closed-loop mechanism is implemented into an open-loop robot structure. High motion tracking performance can thus be achieved by applying relatively simple control algorithms. Following Youcef-Toumi’s design strategy, Diken [9] improved the motion tracking performance of an open-loop robot manipulator by applying a mass redistribution scheme. In his study, the structure of a robot arm is first reduced to dynamically equivalent point masses so as to eliminate the gravitational term in the dynamic model. A simple algorithm is then applied to control the system, and satisfactory trajectory tracking can be obtained.

A more general concept, called ‘design for control’ (DFC), was proposed by the authors elsewhere [10–13]. The essence of the concept is to design the mechanical structure of a programmable machine by fully exploring the physical understanding of the overall system with consideration of the facilitation of controller design as well as the execution of control actions with the least hardware restriction. An intuitive way to realize this objective is to design an appropriate structure for the mechanical part so that it can result in a ‘simple’ dynamic model and thus a more predictable dynamic response [10–12]. Another way is to develop an optimization model in which the design variables for both the mechanical structure and controller are explicitly represented and varied to contribute to a further improvement in system performance.

The work presented in this paper has applied this DFC methodology for improvement of the motion tracking performance of an existing programmable intelligent machine, namely a servomotor-driven, planar, four-bar, closed-loop linkage. Instead of redesigning the mechanical structure for the overall machine system as suggested by Youcef-Toumi [7, 8], synthesis of the mass redistribution of the mechanical structure for an existing four-bar linkage is adopted in this work. Similar to Youcef-Toumi’s idea, the aim of the mass redistribution design is to simplify the dynamic model of the linkage; in contrast to his work, however, the topology of the linkage remains unchanged. Hence, much less design effort is involved at less manufacturing cost. With the simplified dynamic model, the PD control method can show a very good trajectory tracking performance. The work reported here differs from previous work by the present authors [10–12] in the sense that only a PD controller was studied earlier. The work reported here will further show that indeed simultaneous variation in both kinds of design variable, one for mechanical structures and the other for controller structures, can result in further minimization of an objective function in an optimization model.

The mass redistribution design is determined on the basis of the principle of balancing both the shaking force and the shaking moment. It is known that the shaking force and the shaking moment are harmful to the surroundings where a machine is mounted. Balancing of the shaking force and the shaking moment is therefore an important concern in machine design. It will be shown, therefore, that the approach applied here can improve not only the motion tracking performance but also the vibration behaviour of the closed-loop four-bar linkage.

Section 2 of this paper recalls the dynamic model of a four-bar linkage system to aid the reader. Synthesis of the mass redistribution design is presented in Section 3. Section 4 discusses control design. Firstly, a simple PD controller is developed for the system to perform trajectory tracking. Secondly, a promising finding is presented, showing that a concurrent optimization of the structure parameter and control gain can lead to further improvement in system performance. In Section 5, a conclusion is drawn and future work is discussed.

2 DESCRIPTION OF THE FOUR-BAR LINKAGE

For clarity of description, the dynamic model of the four-bar linkage is rewritten here. The details of the derivation can be found in references [14] and [15]. Figure 1 illustrates the configuration of the four-bar
linkage under study. For link \(i\), the location of the centre of mass, denoted by a darkened circle in this figure, is described by variables \(r_i\) and \(\bar{r}_i\), \(m_i\) and \(L_i\) denote the mass and the length of the link, respectively, and \(J_i\) is the moment of inertia with respect to the centroid. In order to obtain a general dynamic model for the four-bar linkage, an effective torsional spring with stiffness constant \(k\) and an effective torsional damper with damping coefficient \(C\) are attached to the follower of the four-bar linkage.

Lagrange's equation is applied to derive the dynamic model of the linkage:

\[
\frac{d}{dt} \frac{\partial K}{\partial \dot{\theta}_i} - \frac{\partial K}{\partial \theta_i} + \frac{\partial P}{\partial \dot{\theta}_i} + \frac{\partial D}{\partial \theta_i} = \tau
\]

where \(K\) is the kinetic energy, \(P\) is the potential energy, \(D\) is the dissipation energy and \(\tau\) is the external input torque. In equation (1), the input crank angle \(\theta_1\) is specified as the generalized coordinate to describe the motion of the four-bar linkage.

The kinetic energy of the linkage system can be described as:

\[
K = \sum_{i=1}^{3} \left[ \frac{1}{2} m_i (V_{ix}^2 + V_{iy}^2) + \frac{1}{2} J_i \dot{\theta}_i^2 \right]
\]

where \(\dot{\theta}_i\) is the angular velocity of link \(i\), and \(V_{ix}\) and \(V_{iy}\) are the \(x\) and \(y\) axis velocity components of the mass centre of link \(i\); \(\dot{\theta}_i\), \(V_{ix}\) and \(V_{iy}\) can be further described as

\[
V_{ix} = u_i \dot{\theta}_1, \quad i = 1, 2, 3
\]

\[
V_{iy} = v_i \dot{\theta}_1, \quad i = 1, 2, 3
\]

\[
\dot{\theta}_i = \omega_i \dot{\theta}_1, \quad i = 1, 2, 3
\]

The detailed expression of \(u_i\), \(v_i\) and \(\omega_i\) can be found in Appendix 1.

Substituting equation (3) into equation (2) gives

\[
K = \frac{1}{2} A(\dot{\theta}_1)^2
\]

with

\[
A(\dot{\theta}_1) = \sum_{i=1}^{3} [m_i(u_i^2 + v_i^2) + J_i \dot{\omega}_i^2]
\]

The potential energy of the mechanism can be expressed as

\[
P = P_s + P_g
\]

where \(P_s\) is the potential energy stored in the torsional spring and \(P_g\) is the potential energy due to gravity. In detail, they are as follows:

\[
P_s = \frac{1}{2} k(\theta_1 - \theta_{3,0})^2
\]

\[
P_g = [m_1 r_1 \sin(\theta_1 + \delta_1) + m_2 L_1 \sin \theta_1 + r_2 \sin(\theta_2 + \delta_2)]\]

\[
+ m_3 L_4 \sin \theta_4 + r_3 \sin(\theta_3 + \delta_3)] g
\]
where $\theta_2, \theta_4, \delta_1, \delta_2$ and $\delta_3$ are illustrated in Fig. 2, $\theta_{3,0}$ is the position of the follower link corresponding to zero deflection of the torsional spring and $g$ is the gravity constant.

The dissipation energy of the system can be described as

$$D = \frac{1}{2} C_\beta \dot{\gamma}_3^2$$

(9)

Substituting the differentiation of equations (5), (7) and (9) into equation (1) and using the relations defined by equation (3) gives

$$A(\theta_1) \ddot{\dot{\theta}}_1 + \frac{1}{2} \frac{dA(\theta_1)}{d\theta_1} \dot{\theta}_1^2 + \{k\omega_3(\theta_1 - \theta_{3,0}) + [m_1r_1 \cos(\theta_1 + \delta_1) + m_2L_1 \cos \theta_1 + m_2r_2 \cos(\theta_2 + \delta_2)\omega_2 + m_3r_3 \cos(\theta_3 + \delta_3)\omega_3][g + C_\beta \dot{\gamma}_1]\} = \tau$$

(10)

where $A(\theta_1)$ is the non-constant generalized inertia coefficient term,

$$\frac{1}{2} \frac{dA(\theta_1)}{d\theta_1}$$

is the Coriolis/centripetal coefficient term, $\{ \cdot \}$ is the total potential energy, $[ \cdot ]$ is the gravity term and the remaining items are the potential energies of the spring and the damper respectively. A full expansion of the above equation is described in Appendix 2.

3 MODIFICATION OF THE FOUR-BAR LINKAGE VIA MASS REDISTRIBUTION

As shown in equation (10), the dynamic model of the four-bar linkage is quite complicated. To design a control algorithm for this system to achieve high performance is not a simple task. Following the DFC concept, this section will present the modification of the mass distribution of the four-bar linkage system, with the aim of simplifying the dynamic model of the mechanism so as to facilitate the controller design. Two approaches have been applied for the redistribution design of the mass of the linkage, namely the complete shaking force balancing method and the partial shaking moment balancing plus complete shaking force balancing method. The shaking force and moment of any machine will cause disturbances such as vibrations, noise, wear and fatigue, and hence limit the full potential of the machine. To balance the shaking force and moment is therefore an essential concern in mechanism design. In this work, dual purposes, i.e. simplification of the dynamic model and disturbance attenuation, can be achieved by balancing the shaking force completely and the shaking moment partially.

3.1 Mass redistribution design by using the complete shaking force balancing scheme

The underlying concept of shaking force balancing is to eliminate the vector sum of the forces acting on the mechanism frame by making the total centre of mass of the mechanism stationary [16]. The shaking force balancing method is illustrated in Fig. 3. The mass centres $S_i$ of the links are located by time-invariant polar
coordinates \((r, \delta)\). Note that \(S_2\) can also be located by the alternative polar coordinate \((r_2', \delta_2')\). To balance the shaking force of the above four-bar mechanism, the following conditions must be satisfied [16]:

\[
\begin{align*}
m_1r_1 &= m_2r_2' L_1 / L_2, \\
m_3r_3 &= m_2r_2 L_3 / L_2,
\end{align*}
\]

\(\delta_1 = \delta_2'
\]

\(\delta_3 = \delta_2 = \pi
\]

(11)

From equation (11) it can be seen that, whenever the mass and the location of the centre of the mass of one of the links are given, the mass distribution of the remaining two links can then be determined. Furthermore, equation (11) can also be applied to determine the size and location of counterweights or negative masses that may need to be added to the mechanism for shaking force elimination.

It is observed from equation (10) that the gravity term in the dynamic model of the closed-loop mechanism is

\[
\begin{align*}
\frac{\partial P}{\partial \theta_1} &= \left[ m_1r_1 \cos(\theta_1 + \delta_1) + m_2L_1 \cos \theta_1 \\
&\quad + m_2r_2 \cos(\theta_2 + \delta_2) \omega_2 \\
&\quad + m_3r_3 \cos(\theta_3 + \delta_3) \omega_3 \right] g
\end{align*}
\]

Substituting equation (11), \(\omega_2, \omega_3\) (given in Appendix 1) and the conditions

\[
\begin{align*}
\theta_3' &= \theta_3 + \pi, \\
r_2' &= \frac{L_2}{\sin(\delta_2')} \sin(\delta_2')
\end{align*}
\]

into the above equation gives

\[
\frac{\partial P}{\partial \theta_1} = 0
\]

(13)

Equation (13) proves that, if the shaking force is completely balanced, the gravitational term will vanish from Lagrange’s equation. Thus, the dynamic model of the mechanism is simplified.

### 3.2 Mass redistribution design by using the partial shaking moment balancing scheme

Since the shaking moment is also harmful to the machine, it should also be balanced. However, it is difficult to design a mass redistribution scheme that could fully balance the shaking force and the shaking moment simultaneously. In this work, mass is redistributed among the linkages following a scheme whereby the shaking moment is partially balanced while the shaking force is fully balanced.

To derive the condition for partial shaking moment balancing, the following equation is presented:

\[
M_k = \sum_{i=1}^{3} K_i \ddot{\theta}_i + K_4(\mu_1 \ddot{\theta}_1 + \mu_1 \ddot{\theta}_1)
\]

(14)

where \(M_k\) is the summation of the input torque and the moment of the ground bearing forces. The parameters in equation (14) can be expressed as [16]

\[
\begin{align*}
K_i &= -m_i(k_i^2 + r_i^2 - L_i r_i \cos \delta_i), \\
K_4 &= -2m_2L_1 \sin \delta_2 \\
\mu_1 &= \frac{L_3}{L_2} \sin(\theta_1 - \theta_3) + \frac{L_4}{L_2} \sin \theta_1 \\
k_i^2 &= \frac{J_i}{m_i}
\end{align*}
\]

(15)

For partial balancing of the shaking moment of the four-bar linkage under the condition of complete shaking force balancing, the mass of coupler link 2 needs to be redistributed so that it is in-line, i.e. \(\delta_2 = 0\) and \(\delta_2' = \pi\). Under this condition, the parameters of equation (15) become

\[
\begin{align*}
K_1 &= -m_1(k_1^2 + r_1^2 + L_1 r_1) \\
K_2 &= -m_2(k_2^2 + r_2^2 - L_2 r_2) \\
K_3 &= -m_3(k_3^2 + r_3^2 + L_3 r_3) \\
K_4 &= 0
\end{align*}
\]

(16)

From equation (16) it is observed that the shaking moment of coupler link 2 will be eliminated only if

\[
k_2^2 = r_2 r_2'
\]

(17)

where \(k_2^2 = r_2 r_2'\).

### 3.3 Result presentation

The analysis in the previous sections presents the following results:

1. To obtain complete shaking force balancing, equation (11) must be satisfied.
2. To achieve partial shaking moment balancing and complete shaking force balancing, equations (11) and (17) must be satisfied simultaneously.

The parameters of the four-bar linkage under different situations are recorded in Table 1. Case 1 describes the linkage shown in Fig. 1, where neither the shaking force nor the shaking moment balancing is considered. Case 2
shows the parameters of the modified linkage with the shaking force being completely eliminated. The parameters of the linkage where the complete shaking force and the partial shaking moment are balanced are shown in Case 3.

It is noted from Table 1 that the negative mass distribution approach is adopted for modification of the linkage dynamics, as this approach has an advantage in reducing the system weight and hence the inertia term of the system. After the modification schemes are applied, the dynamic model of the linkage given in equation (10) is simplified as

\[
A(\dot{\theta}_1)\ddot{\theta}_1 + \frac{1}{2} \frac{dA(\dot{\theta}_1)}{d\dot{\theta}_1} \dot{\theta}_1^2 = \tau \tag{18}
\]

where the spring constant \( k \) and the damping coefficient \( C \) are assumed to be zero in all three cases.

### 4 CONTROL ALGORITHM DESIGN

#### 4.1 PD controller design and simulation result

Once the gravitational term is eliminated by structure design, the ‘workload’ to a control algorithm is thus reduced. For example, no control effort needs to be made to deal with the gravitational term which is otherwise a non-trivial task often leading to a complex control algorithm (see reference [3]). In the following, a simple PD control algorithm is developed:

\[
\tau(t) = K_p \theta_e(t) + K_d \dot{\theta}_e(t) \tag{19}
\]

where \( \tau(t) \) is the driving torque generated by the controller. \( K_p \) and \( K_d \) are the proportional and derivative gains respectively, \( \theta_e(t) = \theta_1^d(t) - \theta_1(t) \) represents the angular trajectory error appearing in the input crank, with \( \theta_1^d(t) \) and \( \theta_1(t) \) as the desired and actual angular displacements respectively, and \( \dot{\theta}_1(t) \) is the angular velocity error of the input crank.

Simulation studies were carried out for the linkages of the three cases. Assume that the input crank rotates at a high-speed constant velocity of 30 rad/s. Note that this a speed that conventional open-loop serial manipulator systems could not achieve. Since the behaviour of the system is highly non-linear, the conventional linear control methods used to design control gains are not applicable. The following performance index, \( I \), is used for the selection of the PD gains:

\[
I = \sum_{t=0}^{t=t_f} \dot{\theta}_e^2 + \sum_{t=0}^{t=t_f} \ddot{\theta}_e^2 + Z \sum_{t=0}^{t=t_f} \tau^2 \tag{20}
\]

where \( Z \) is a weighted constant. Here, \( Z \) is chosen to be 0.001 in order to keep \( \sum_{t=0}^{t=t_f} \tau^2 \) in the same order as the other two terms. The control gains were selected to achieve the minimal performance indexes. A Matlab/ Simulink software package was used for the above purpose.

Table 2 shows the optimal control gains for these three cases. Some other results based on the simulation are illustrated in Fig. 4, where the solid lines indicate the results of case 1, i.e. the original linkage without any modification; the dash-dotted lines indicate the results of case 2, i.e. the linkage with completely balanced shaking force; the dotted lines represent the results of case 3, where shaking force is completely balanced and shaking moment is partially balanced.

A comparative analysis based on the simulation results clearly shows that, after applying the mass redistribution schemes, the motion tracking performance of the system is improved significantly. As shown in Fig. 4a, the gravitational terms for cases 2 and 3 vanish as anticipated. Figure 4b shows that the constant-velocity tracking error is reduced in case 2, and is further reduced in case 3. Figure 4c shows that the angular displacement tracking performance is improved in case 2, and a better result is obtained in case 3. Figure 4d shows the torque profile, which implies that less control energy is consumed in cases 2 and 3. Furthermore, the effectiveness of using the negative mass redistribution approach is reflected by the profiles presented in Figs 4e and f respectively, where the inertia

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_1 ) (m)</td>
<td>0.1020</td>
<td>0.1020</td>
<td>0.1020</td>
</tr>
<tr>
<td>( L_2 ) (m)</td>
<td>0.6100</td>
<td>0.6100</td>
<td>0.6100</td>
</tr>
<tr>
<td>( L_3 ) (m)</td>
<td>0.4060</td>
<td>0.4060</td>
<td>0.4060</td>
</tr>
<tr>
<td>( L_4 ) (m)</td>
<td>0.5993</td>
<td>0.5993</td>
<td>0.5993</td>
</tr>
<tr>
<td>( r_1 ) (m)</td>
<td>0.0000</td>
<td>0.0944</td>
<td>0.0981</td>
</tr>
<tr>
<td>( r_2 ) (m)</td>
<td>0.3050</td>
<td>0.0457</td>
<td>0.0236</td>
</tr>
<tr>
<td>( r_3 ) (m)</td>
<td>0.2030</td>
<td>0.2030</td>
<td>0.1048</td>
</tr>
<tr>
<td>( m_1 ) (kg)</td>
<td>1.3620</td>
<td>0.6810</td>
<td>0.6810</td>
</tr>
<tr>
<td>( m_2 ) (kg)</td>
<td>1.3620</td>
<td>0.6810</td>
<td>0.6810</td>
</tr>
<tr>
<td>( m_3 ) (kg)</td>
<td>0.2041</td>
<td>0.1021</td>
<td>0.1021</td>
</tr>
<tr>
<td>( J_1 ) (kgm²)</td>
<td>0.0131</td>
<td>0.0010</td>
<td>0.0000</td>
</tr>
<tr>
<td>( J_2 ) (kgm²)</td>
<td>0.1173</td>
<td>0.0257</td>
<td>0.0094</td>
</tr>
<tr>
<td>( J_3 ) (kgm²)</td>
<td>0.0051</td>
<td>0.0026</td>
<td>0.0031</td>
</tr>
<tr>
<td>( \dot{\theta}_1 ) (rad)</td>
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<td>( \pi )</td>
<td>( \pi )</td>
</tr>
<tr>
<td>( \dot{\theta}_2 ) (rad)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \dot{\theta}_3 ) (rad)</td>
<td>0</td>
<td>( \pi )</td>
<td>( \pi )</td>
</tr>
<tr>
<td>( \dot{\theta}_4 ) (rad)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( k )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( C )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2 Optimized gains for the three cases

<table>
<thead>
<tr>
<th>Values</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of ( K_p ) optimized</td>
<td>12.9836</td>
<td>10.0870</td>
<td>10.0352</td>
</tr>
<tr>
<td>Value of ( K_i ) optimized</td>
<td>6.8592</td>
<td>9.6523</td>
<td>9.998</td>
</tr>
<tr>
<td>Performance index</td>
<td>327.7990</td>
<td>134.2495</td>
<td>124.5452</td>
</tr>
</tbody>
</table>
Fig. 4 (continued over)
Fig. 4 Tracking performance of the four-bar linkage: (a) profiles of gravity terms; (b) profiles of angular velocity tracking errors; (c) profiles of angular displacement tracking errors; (d) profile of torque; (e) profiles of inertia terms; (f) profiles of Coriolis/centrifugal terms.

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- for Case 1: original linkage;
- for Case 2: linkage with balanced shaking force;
- for Case 3: linkage with balanced shaking force and partially balanced shaking moment.
and Coriolis/centrifugal terms are shown to be reduced significantly in cases 2 and 3.

4.2 Simultaneous change in the structure parameter and the control gain

The possibility of further improving the system performance by simultaneously varying the structure parameters is studied in this work on the basis of simulation. The simulation scheme is as follows:

(a) to vary the length of certain links,
(b) to optimize the control gains (the PD control algorithm is again used),
(c) to compute the performance index defined in equation (20).

In the simulation study, it is assumed that full shaking force balancing remains valid. In practice, this would require the mass of the system to be appropriately designed to meet the conditions represented in equation (11).

Table 3 shows the result of changing the length of link 2 (i.e. \(L_2\)). It is clear from the table that between simulation case 11 and case 12 there is a minimal performance index. Table 4 further shows the result of changing the length of both link 4 and link 2. It is clear from the table that between simulation case 22 and case 23 and between simulation case 23 and case 24 there must be a minimum in terms of the performance index.

5 CONCLUSION AND DISCUSSION

In this paper some further results are brought forward from previous work reported in references [10] to [12]. It has been verified through simulation that the trajectory tracking error and torque fluctuation are further reduced with a simple PD control method, combined with mass reconfiguration based on a scheme of full shaking force balancing and partial shaking moment balancing. The implementation of mass reconfiguration through a so-called negative mass approach appears to be very effective, with, however, a possible restriction of ‘reducing’ the mass from a practical point of view. This practical approach should allow for both strength and stiffness. It is also shown that a design model based on simultaneous optimization of both link length and control gain is feasible.

Note that, for the mechanism system presented by Tao and Sadler [2], where the mechanical structure is determined in advance of the controller design, a simple PD controller results in a dynamic performance with high vibration. In order to reduce the vibration behaviour, Lin and Chen propose a special disturbance compensation control algorithm in their work [3], with several other complicated control loops designed for motion tracking purposes. Let an overall control algorithm be considered as a combination of some special control algorithms (perhaps called elementary control algorithms). It is believed that these different elementary control algorithms may not create a consistent or harmonious overall control algorithm. That is to say, it may be possible that one elementary control algorithm, say CA, which is specially designed to tackle one particular problem, say PA, has a negative effect on another

### Table 3 Simultaneous change in the length of link 2 and the control gain

<table>
<thead>
<tr>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
<th>Case 8</th>
<th>Case 9</th>
<th>Case 10</th>
<th>Case 11</th>
<th>Case 12</th>
<th>Case 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_1)</td>
<td>0.104</td>
<td>0.103</td>
<td>0.102</td>
<td>0.101</td>
<td>0.101</td>
<td>0.1006</td>
<td>0.1003</td>
<td>0.1001</td>
<td>0.0999</td>
</tr>
<tr>
<td>(L_2)</td>
<td>0.41</td>
<td>0.51</td>
<td>0.61</td>
<td>0.71</td>
<td>0.81</td>
<td>0.91</td>
<td>1.04</td>
<td>1.11</td>
<td>1.21</td>
</tr>
<tr>
<td>(L_3)</td>
<td>0.273</td>
<td>0.333</td>
<td>0.406</td>
<td>0.473</td>
<td>0.539</td>
<td>0.6061</td>
<td>0.6726</td>
<td>0.7393</td>
<td>0.8059</td>
</tr>
<tr>
<td>(L_4)</td>
<td>0.50</td>
<td>0.5593</td>
<td>0.5593</td>
<td>0.5593</td>
<td>0.5593</td>
<td>0.5593</td>
<td>0.5593</td>
<td>0.5593</td>
<td>0.5593</td>
</tr>
<tr>
<td>Performance index</td>
<td>128.5424</td>
<td>125.7149</td>
<td>124.5452</td>
<td>123.2391</td>
<td>123.1723</td>
<td>122.5711</td>
<td>122.1465</td>
<td>121.9269</td>
<td>121.9465</td>
</tr>
</tbody>
</table>

### Table 4 Simultaneous change in the lengths of link 4 and link 2 and the control gain

<table>
<thead>
<tr>
<th>Case 17</th>
<th>Case 18</th>
<th>Case 19</th>
<th>Case 20</th>
<th>Case 21</th>
<th>Case 22</th>
<th>Case 23</th>
<th>Case 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_1)</td>
<td>0.1001</td>
<td>0.1001</td>
<td>0.1001</td>
<td>0.1001</td>
<td>0.0997</td>
<td>0.0997</td>
<td>0.0997</td>
</tr>
<tr>
<td>(L_2)</td>
<td>1.11</td>
<td>1.11</td>
<td>1.11</td>
<td>1.11</td>
<td>1.11</td>
<td>1.11</td>
<td>1.11</td>
</tr>
<tr>
<td>(L_3)</td>
<td>0.7393</td>
<td>0.7393</td>
<td>0.7393</td>
<td>0.7393</td>
<td>0.7393</td>
<td>0.9391</td>
<td>0.9391</td>
</tr>
<tr>
<td>(L_4)</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.8</td>
<td>0.9</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>Performance index</td>
<td>121.2060</td>
<td>120.4402</td>
<td>120.0798</td>
<td>119.8660</td>
<td>119.7372</td>
<td>119.4305</td>
<td>121.7803</td>
</tr>
</tbody>
</table>
particular problem, say PB. Therefore, philosophically, improvement in the system performance has an upper limit. A careful design of system structures aiming to eliminate some problems that would otherwise be dealt with by control algorithms may further raise this upper limit.

ACKNOWLEDGEMENTS

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REFERENCES


APPENDIX 1

The terms \( u_i, v_i \) and \( \omega_i \) can be described as follows [14]:

\[
\omega_i = \frac{\dot{\theta}_i}{\dot{\theta}_1} = \frac{L_1 \sin(\theta_i - \theta_1)}{L_i \sin(\theta_i - \theta_j)}
\]

where \( i \) and \( j \) are any cyclic permutation of 2 and 3;

\[
\begin{bmatrix}
u_i \\
v_j
\end{bmatrix} = \begin{bmatrix}
u_R \\
v_R
\end{bmatrix} + \begin{bmatrix}x_i \theta \end{bmatrix} \omega_i
\]

where \( R \) represents points A, B and D of links 1, 2 and 3 respectively.

The detailed expressions of \( u_i, v_i \) and \( \omega_i \) are given as [14]

\[
\omega_1 = 1
\]

\[
\omega_2 = \frac{L_1 \sin(\theta_1' - \theta_1)}{L_2 \sin(\theta_2 - \theta_1')}
\]

\[
\omega_3 = \frac{L_1 \sin(\theta_2 - \theta_1)}{L_3 \sin(\theta_3' - \theta_2)}
\]

\[
\begin{bmatrix}
u_1 \\
v_1
\end{bmatrix} = \begin{bmatrix} -\xi_1 S_1 - \eta_1 C_1 \\
\xi_1 C_1 - \eta_1 S_1
\end{bmatrix}
\]

\[
\begin{bmatrix}
u_2 \\
v_2
\end{bmatrix} = \begin{bmatrix} -L_1 S_1 \\
L_1 C_1
\end{bmatrix} + \begin{bmatrix} -\xi_2 S_2 - \eta_2 C_2 \\
\xi_2 C_2 - \eta_2 S_2
\end{bmatrix} \omega_2
\]

\[
\begin{bmatrix}
u_3 \\
v_3
\end{bmatrix} = \begin{bmatrix} -\xi_3 S_3 - \eta_3 C_3 + L_3 S_1 \\
\xi_3 C_3 - \eta_3 S_3 - L_3 C_3
\end{bmatrix} \omega_3
\]

where \( \theta_1' = \theta_3 + \pi, C_i = \cos \theta_i \) and \( S_i = \sin \theta_i \). Note that \( \xi_i, \eta_i \) and the local coordinate of each link are shown in Fig. 2. Furthermore, the coordinates of point \( P_i \) on link \( i \)
can be determined with reference to the local coordinate system:

\[ x_{IR} = x_i - x_R = \xi_i \cos \theta_i - \eta_i \sin \theta_i \]
\[ y_{IR} = y_i - y_R = \xi_i \sin \theta_i + \eta_i \cos \theta_i \]

**APPENDIX 2**

Term \( A(\theta_1) \) in equation (10) can be expressed in the more compact form

\[ A(\theta_1) = C_0 + C_1\omega_2^2 + C_2\omega_3^2 + C_3\cos(\theta_2 - \theta_1 + \delta_2) \]

where \( C_i \) (\( i = 0, 1, 2, 3 \)) are coefficients containing the parameters for mass distribution.

The coefficients \( C_i \) (\( i = 0, 1, 2, 3 \)) are given as [14]

\[ C_0 = J_1 + m_1\xi_1^2 + m_2L_1^2 \]
\[ C_1 = J_2 + m_2\eta_2^2 \]
\[ C_2 = J_3 + m_3\eta_3^2 \]
\[ C_3 = 2m_2\xi_2L_1 \]

The expression of \( [dA(\theta_1)]/d\theta_1 \) in equation (10) can be written as

\[
\frac{dA(\theta_1)}{d\theta_1} = 2C_1\omega_2 \frac{d\omega_2}{d\theta_1} + 2C_2\omega_3 \frac{d\omega_3}{d\theta_1} + C_3 \times \left[ \frac{d\omega_2}{d\theta_1} \cos(\theta_2 - \theta_1 + \delta_2) - \omega_2 \sin(\theta_2 - \theta_1 + \delta_2)(\omega_2 - 1) \right]
\]

where

\[
\frac{d\omega_2}{d\theta_1} = \frac{L_1(D_1 + D_2)}{L_2 \sin^2(\theta_2 - \theta_3)}
\]
\[
\frac{d\omega_3}{d\theta_1} = \frac{L_1(D_3 + D_4)}{L_3 \sin^2(\theta_2 - \theta_3)}
\]
\[ D_1 = (\omega_3 - 1) \sin(\theta_2 - \theta_3) \cos(\theta_3 - \theta_1) \]
\[ D_2 = \sin(\theta_3 - \theta_1) \cos(\theta_2 - \theta_3)(\omega_3 - \omega_2) \]
\[ D_3 = (\omega_2 - 1) \sin(\theta_2 - \theta_3) \cos(\theta_2 - \theta_1) \]
\[ D_4 = \sin(\theta_2 - \theta_1) \cos(\theta_2 - \theta_3)(\omega_3 - \omega_2) \]